2.12.1 Practicals Question Bank

Linear Algebra

Unit-I

1. Let H be the set of all vectors of the form $\begin{bmatrix} -2t \\ 5t \\ 3t \end{bmatrix}$. Find a vector $\mathbf{v}$ in $\mathbb{R}^3$ such that $H = Span\{\mathbf{v}\}$.Why does this show that H is a subspace of $\mathbb{R}^3$?

2. Let V be the first quadrant in the xy-plane; that is, let $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x \geq 0, y \geq 0 \right\}$

   a. If $\mathbf{u}$ and $\mathbf{v}$ are in V, is $\mathbf{u} + \mathbf{v}$ in V? Why?

   b. Find a specific vector $\mathbf{u}$ in V and a specific scalar c such

3. Let $\mathbf{v_1} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ and $\mathbf{v_2} = \begin{bmatrix} -2 \\ 7 \\ -9 \end{bmatrix}$. Determine if $\{\mathbf{v_1}, \mathbf{v_2}\}$ is a basic for $\mathbb{R}^3$. Is $\{\mathbf{v_1}, \mathbf{v_2}\}$ a basis for $\mathbb{R}^2$?

4. The set $\mathcal{B} = \{1 + t^2, t + t^2, 1 + 2t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\mathbf{p}(t) = 1 + 4t + 7t^2$ relative to $\mathcal{B}$.

5. The set $\mathcal{B} = \{1 - t^2, t - t^2, 2 - t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\mathbf{p}(t) = 1 + 3t - 6t^2$ relative to $\mathcal{B}$.

6. The vector $\mathbf{v_1} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$, $\mathbf{v_3} = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$ $span \mathbb{R}^2$ but do not form a basis. Find two different ways to express $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as a linear combination of $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$.

7. Find the dimension of the subspace of all vectors in $\mathbb{R}^3$ whose first and third entries are equal.

8. Find the dimension of the subspace H of $\mathbb{R}^2$ spanned by $\begin{bmatrix} 1 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 10 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 15 \end{bmatrix}$.

9. Let H be an n-dimensional subspace of an n-dimensional vector space V. Show that $H = V$.

10. Explain why the space $\mathbb{P}$ of all polynomials is an infinite dimensional space.

Unit-II

11. If a 4×7 matrix A has rank 3, find dim Nul A, dim Row A, and rank A^T.

12. If a 7×5 matrix A has rank 2, find dim Nul A, dim Row A, and rank A^T.

13. If the null space of an 8×5 matrix A is 3-dimensional, what is the dimension of the row space of A?

14. If A is a 3×7 matrix, what is the smallest possible dimension of Nul A?

15. Let $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Find $\mathbf{v}$ in $\mathbb{R}^3$ such that $\begin{bmatrix} 1 & -3 & 4 \\ 2 & -6 & 8 \end{bmatrix} = \mathbf{u}\mathbf{v}^T$.

16. If A is a 7×5 matrix, what is the largest possible rank of A? If A is a 5×7 matrix, what is the largest possible rank of A? Explain your answers.

17. Without calculations, list rank A and dim Nul A

$$A = \begin{bmatrix} 2 & 6 & -6 & 6 & 3 & 6 \\ -2 & -3 & 6 & -3 & 0 & -6 \\ 4 & 9 & -12 & 9 & 3 & 12 \\ -2 & 3 & 6 & 3 & 3 & -6 \end{bmatrix}.$$

18. Use a property of determinants to show that A and A^T have the same characteristic polynomial.

19. Find the characteristic equation of

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

.

20. Find the characteristic polynomial and the real eigenvalues of $\begin{bmatrix} 4 & 0 & -1 \\ 0 & 4 & -1 \\ 1 & 0 & 2 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

Unit-III

21. Let $A = PDP^{-1}$ and compute A^4, where $P = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$,$D = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

22. Let $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}\}$ and $\mathcal{D} = \{\mathbf{d_1}, \mathbf{d_2}\}$ be bases for vector spaces V and W, respectively. Let $T : V \to W$ be a linear transformation with the property that $T(\mathbf{b_1}) = 3\mathbf{d_1} - 5\mathbf{d_2}$, $T(\mathbf{b_2}) = -\mathbf{d_1} + \mathbf{6d_2}$, $T(\mathbf{b_3}) = 4\mathbf{d_2}$. Find the matrix for T relative to $\mathcal{B}$ and $\mathcal{D}$.

23. Let $\mathcal{D} = \{\mathbf{d_1}, \mathbf{d_2}\}$ and $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}\}$ be bases for vector spaces V and W, respectively. Let $T : V \to W$ be a linear transformation with the property that $T(\mathbf{d_1}) = 3\mathbf{b_1} - 3\mathbf{b_2}$, $T(\mathbf{d_2}) = -\mathbf{2b_1} + \mathbf{5b_2}$. Find the matrix for T relative to $\mathcal{B}$ and $\mathcal{D}$.

24. Let $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}\}$ be a basis for a vector space V and let $T : V \to \mathbb{R}^2$ be a linear transformation with the property that

$$T(x_1\mathbf{b_1} + x_2\mathbf{b_2} + x_3\mathbf{b_3}) = \begin{bmatrix} 2x_1 - 3x_2 + x_3 \\ -2x_1 + 5x_3 \end{bmatrix}$$

Find the matrix for T relative to $\mathcal{B}$ and the standard basis for $\mathbb{R}^2$.

25. Let $T : \mathbb{P}_2 \to \mathbb{P}_3$ be the transformation that maps a polynomial $\mathbf{p}(t)$ into the polynomial $(t + 3)\mathbf{p}(t)$.

a. Find the image of $\mathbf{p}(t) = 3 - 2t + t^2$.

b. Show that T is a linear transformation.

c. Find the matrix for T relative to the bases $\{1, t, t^2\}$ and $\{1, t, t^2, t^3\}$.

26. Assume the mapping $T : \mathbb{P}_2 \to \mathbb{P}_2$ defined by $T(a_0+a_1t+a_2t^2) = 3a_0+(5a_0-2a_1)t+(4a_1+a_2)t^2$ is linear. Find the matrix representation of T relative to the basis $\mathcal{B} = \{1, t, t^2\}$.

27. Define $T : \mathbb{P}_3 \to \mathbb{R}^4$ by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(-2) \\ \mathbf{p}(3) \\ \mathbf{p}(1) \\ \mathbf{p}(0) \end{bmatrix}$

a. Show that T is a linear transformation.

b. Find the matrix for T relative to the bases $\{1, t, t^2, t^3\}$ for $\mathbb{P}_3$ and the standard basis for $\mathbb{R}^4$.

28. Let A be a 2×2 matrix with eigenvalues -3 and -1 and corresponding eigenvectors $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Let $\mathbf{x}(t)$ be the position of a particle at time t. Solve the initial value problem $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

29. Construct the general solution of $\mathbf{x}' = A\mathbf{x}$. $A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} -7 & 10 \\ -4 & 5 \end{bmatrix}$

30. Compute the orthogonal projection of $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$ onto the line through $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ and the origin.

2.13.1 Practicals Question Bank

Solid Geometry

Unit-I

1. Find the equation of the sphere through the four points $(4, -1, 2), (0, -2, 3), (1, -5, -1), (2, 0, 1)$.

2. Find the equation of the sphere through the four points $(0, 0, 0), (-a, b, c), (a, -b, c), (a, b, -c)$.

3. Find the centre and the radius of the circle $x + 2y + 2 = 15, x^2 + y^2 + z^2 - 2y - 4z = 11$.

4. Show that the following points are concyclic:

   (i) $(5, 0, 2), (2, -6, 0), (7, -3, 8), (4, -9, 6)$.

   (ii) $(-8, 5, 2), (-5, 2, 2)(-7, 6, 6), (-4, 3, 6)$.

5. Find the centres of the two spheres which touch the plane $4x + 3y + = 47$ at the points $(8, 5, 4)$ and which touch the sphere $x^2 + y^2 + z^2 = 1$.

6. Show that the spheres
$$x^2 + y^2 + z^2 = 25$$
$$x^2 + y^2 + z^2 - 24x - 40y - 18z + 225 = 0$$
touch externally and find the point of the contact.

7. Find the equation of the sphere that passes through the two points $(0, 3, 0), (-2, -1, -4)$ and cuts orthogonally the two spheres
$$x^2 + y^2 + z^2 + x - 3z - 2 = 0, 2(x^2 + y^2 + z^2) + x + 3y + 4 = 0.$$

8. Find the limiting points of the co-axal system of spheres
$$x^2 + y^2 + z^2 - 20x + 30y - 40z + 29 + \lambda(2x - 3y + 4z) = 0.$$

9. Find the equation to the two spheres of the co-axal systems
$$x^2 + y^2 + z^2 - 5 + \lambda(2x + y + 3z - 3) = 0,$$
which touch the plane
$$3x + 4y = 15.$$

10. Show that the radical planes of the sphere of a co-axal system and of any given sphere pass through a line.

Unit-II

11. Find the equation of the cone whose vertex is the point $(1, 1, 0)$ and whose guiding curve is
$$y = 0, x^2 + z^2 = 4.$$

12. The section of a cone whose vertex is P and guiding curve the ellipse $x^2/a^2 + y^2/b^2 = 1, z = 0$ by the plane $x = 0$ is a rectangular hyperbola. Show that the locus of P is

$$\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1.$$

13. Find the enveloping cone of the sphere

$$x^2 + y^2 + z^2 - 2x + 4z = 1$$

with its vertex at $(1, 1, 1)$.

14. Find the equation of the quadric cone whose vertex is at the origin and which passes through the curve given by the equations

$$ax^2 + by^2 + cz^2 = 1, lx + my + nz = p.$$

15. Find the equation of the cone with vertex at the origin and direction cosines of its generators satisfying the relation

$$3l^2 - 4m^2 + 5n^2 = 0.$$

16. Find the equation of the cylinder whose generators are parallel to

$$x = -\frac{1}{2}y = \frac{1}{3}z$$

and whose guiding curve is the ellipse

$$x^2 + 2y^2 = 1, z = 3.$$

17. Find the equation of the right circular cylinder of radius 2 whose axis is the line

$$\frac{(x-1)}{2} = (y-2) = \frac{(z-3)}{2}.$$

18. The axis of a right circular cylinder of radius 2 is

$$\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1};$$

show that its equation is

$$10x^2 + 5y^2 + 13z^2 - 12xy - 6yz - 4zx - 8x + 30y - 74z + 59 = 0.$$

19. Find the equation of the circular cylinder whose guiding circle is

$$x^2 + y^2 + z^2 - 9 = 0, x - y + z = 3.$$

20. Obtain the equation of the right circular cylinder described on the circle through the three points $(1, 0, 0), (0, 1, 0), (0, 0, 1)$ as guiding circle.

Unit-III

21. Find the points of intersection of the line

$$-\frac{1}{3}(x+5) = (y-4) = \frac{1}{7}(z-11)$$

with the conicoid

$$12x^2 - 17y^2 + 7z^2 = 7.$$

22. Find the equations to the tangent planes to

$$7x^2 - 3y^2 - z^2 + 21 = 0,$$

which pass through the line,

$$7x - 6y + 9 = 3, z = 3.$$

23. Obtain the tangent planes to the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

which are parallel to the plane

$$lx + my + nz = 0.$$

24. Show that the plane $3x + 12y - 6z - 17 = 0$ touches the conicoid $3x^2 - 6y^2 + 9z^2 + 17 = 0$, and find the point of contact.

25. Find the equations to the tangent planes to the surface

$$4x^2 - 5y^2 + 7z^2 + 13 = 0,$$

parallel to the plane

$$4x + 20y - 21z = 0.$$

Find their points of contact also.

26. Find the locus of the perpendiculars from the origin to the tangent planes to the surface

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

which cut off from its axes intercepts the sum of whose reciprocals is equal to a constant $1/k$.

27. If the section of the enveloping cone of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

whose vertex is P by the plane $z = 0$ is a rectangular hyperbola, show that the locus of P is

$$\frac{x^2 + y^2}{a^2 + b^2} + \frac{z^2}{c^2} = 1.$$

28. Find the locus of points from which three mutually perpendicular tangent lines can be drawn to the conicoid $ax^2 + by^2 + cz^2 = 1$.

29. $P(1, 3, 2)$ is a point on the conicoid,

$$x^2 - 2y^2 + 3z^2 + 5 = 0.$$

Find the locus of the mid-points of chords drawn parallel to OP.

2.14.1 Practicals Question Bank

Integral Calculus

Unit-I

1. Let $R = [-3,3] \times [-2,2]$. Without explicitly evaluating any iterated integrals, determine the value of
$$\iint_R (x^5 + 2y)dA$$
2. Integrate the function $f(x,y) = 3xy$ over the region bounded by $y = 32x^3$ and $y = \sqrt{x}$.
3. Integrate the function $f(x,y) = x+y$ over the region bounded by $x+y = 2$ and $y^2 - 2y - x = 0$.
4. Evaluate $\iint_D xydA$, where D is the region bounded by $x = y^3$ and $y = x^2$.
5. Evaluate $\iint_D e^{x^2} dA$, where D is the triangular region with vertices $(0,0)$, $(1,0)$ and $(1,1)$.
6. Evaluate $\iint_D 3ydA$, where D is the region bounded by $xy^2 = 1$, $y = x$, $x = 0$ and $y = 3$.
7. Evaluate $\iint_D (x - 2y)dA$, where D is the region bounded by $y = x^2 + 2$ and $y = 2x^2 - 2$.
8. Evaluate $\iint_D (x^2 + y^2)dA$, where D is the region in the first quadrant bounded by $y = x$, $y = 3x$ and $xy = 3$.
9. Consider the integral
$$\int_0^2 \int_{x^2}^{2x} (2x+1)\, dy\, dx$$
   a) Evaluate this integral.
   b) Sketch the region of integration.
   c) Write an equivalent iterated integral with the order of integration reverse. Evaluate this new integral and check your answer agrees with part (a).
10. Find the volume of the region under the graph of
$$f(x,y) = 2 - |x| - |y|$$
and above the $xy-$plane

Unit-II

Integrate the following over the indicated region W.

11. $f(x,y,z) = 2x - y + z$; W is the region bounded by the cylinder $z = y^2$, the $xy-$plane, the planes $x = 0$, $x = 1$, $y = -2$, $y = 2$.
12. $f(x,y,z) = y$; W is the region bounded by the plane $x + y + z = 2$, the cylinder $x^2 + z^2 = 1$ and $y = 0$.
13. $f(x,y,z) = 8xyz$; W is the region bounded by the cylinder $y = x^2$, the plane $y + z = 9$ and the $xy-$plane.

14. $f(x, y, z) = z$; W is the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and the planes $y = x$,$x = 0$ and $z = 0$.

15. $f(x, y, z) = 1 - z^2$; W is the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$,$(0, 2, 0)$ and $(0, 0, 3)$.

16. $f(x, y, z) = 3x$; W is the region in the first octant bounded by $z = x^2 + y^2$,$x = 0$,$y = 0$ and $z = 4$.

17. $f(x, y, z) = x + y$; W is the region bounded by the cylinder $x^2 + 3z^2 = 9$ and the plane $y = 0$, $x + y = 3$.

18. $f(x, y, z) = z$; W is the region bounded by $z = 0$, $x^2 + 4y^2 = 4$ and $z = x + 2$.

Unit-III

Integrate the following over the indicated region W.

19. $f(x, y, z) = 4x + y$; W is the region bounded by $x = y^2$, $y = z$, $x = y$ and $z = 0$.

20. $f(x, y, z) = x$; W is the region in the first octant bounded by $z = x^2 + 2y^2$, $z = 6 - x^2 - y^2$, $x = 0$ and $y = 0$.

Let $T(u, v) = (3u, -v)$

21. Write $T(u, v)$ as $A \begin{bmatrix} u \\ v \end{bmatrix}$ for a suitable matrix A.

22. Describe the image $D = T(D^*)$, where D^* is the unit square $[0, 1] \times [0, 1]$.

23. Determine the value of

$$\iint_D \sqrt{\frac{x+y}{x-2y}}\,dA$$

where D is the region in $\mathbf{R}^2$ enclosed by the lines $y = \frac{x}{2}$, $y = 0$ and $x + y = 1$.

24. Evaluate

$$\iint_D \sqrt{\frac{(2x+y-3)^2}{(2y-x+6)^2}}\,dxdy.$$

where D is the square with vertices $(0, 0)$, $(2, 1)$, $(3, -1)$ and $(1, -2)$.(Hint: First sketch D and find the equations of its sides.)

25. Evaluate

$$\iint_D \cos(x^2 + y^2)\,dA.$$

where D is the shaded region in the following figure-1.

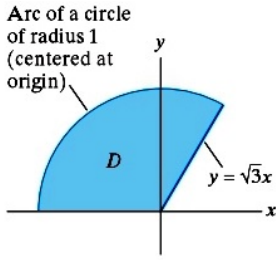

Figure 1:

26. Evaluate
$$\iint\limits_{D} \frac{1}{\sqrt{4-x^2-y^2}}dA.$$
Where D is the disk of radius 1 with center at $(0,1)$.(Be careful when you describe D.)

27. Determine the value of
$$\iiint\limits_{W} \frac{z}{\sqrt{x^2+y^2}}dV.$$
where W is the solid region bounded by the plane $z = 12$ and the paraboloid $z = 2x^2+2y^2-6$.
